

# Earthquake response of bridges with energy dissipators and based isolators: A finite element approach

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**ABSTRACT:** Two finite element (FE) based procedures or models are presented for evaluating earthquake response of bridges with energy dissipators and base isolators. Realistic representation of nonlinear and hysteretic type force-deformation characteristics of the base isolator is emphasized in the models. Piers are assumed to be fixed to the ground in the first model, while effects of soil-structure interaction are taken into account in the second model. A post processor is developed to express the FE results in spectral response form. The significance of base isolation and soil-structure interaction effects on the system response is discussed.

## 1 INTRODUCTION

Earthquake resistant design of bridges requires realistic analytical or numerical procedures for evaluating peak forces induced by intense ground vibrations. The existing aseismic design approaches for bridges can be divided into two broad categories: (i) ductile design approach, and (ii) base isolation approach. The first approach requires selected structural components to yield and deform in a ductile manner during a credible earthquake. The structural components are given a form which may not be optimum either for load bearing or for energy dissipation role. As a result, bridges designed according to this approach may suffer extensive damages when design earthquakes are exceeded (Gates, 1979).

In the base isolation approach, on the other hand, the fundamental period is made sufficiently long so that the structure is isolated from the major disturbing motions at the likely predominant ground motion frequencies. The isolation is done by mounting the bridge deck on laterally flexible bearings. Because earthquakes often transmit energy at several frequencies, it is likely that some frequency component of the ground motion may be in resonance with some structural components resulting in excessive deflections.

This situation is brought under control by introducing artificial energy absorbing devices in the system. In fact, base isolated bridges are always used in conjunction with energy absorbing devices (also called energy dissipators). As the deck is effectively isolated from severe ground motions, the response remains essentially elastic and the main structural components do not suffer any permanent damage.

## 2 SCOPE

The main objective of this paper is to present two analysis procedures, based on finite element (FE) technique, for evaluating earthquake response of bridges with energy dissipators and base isolators. In FE modeling, emphasis is given to represent realistically the force-deformation characteristics of the energy dissipator and base isolator. The first model assumes that the piers are fixed at the base (see Fig. 1) and, therefore, does not account for possible interaction between the deck-pier system and the supporting soil medium. The second model includes the soil-structure interaction effects by simultaneously idealizing the structure and the soil medium. Comparison of results obtained from the two models, demonstrates the significance of soil-structure inter-

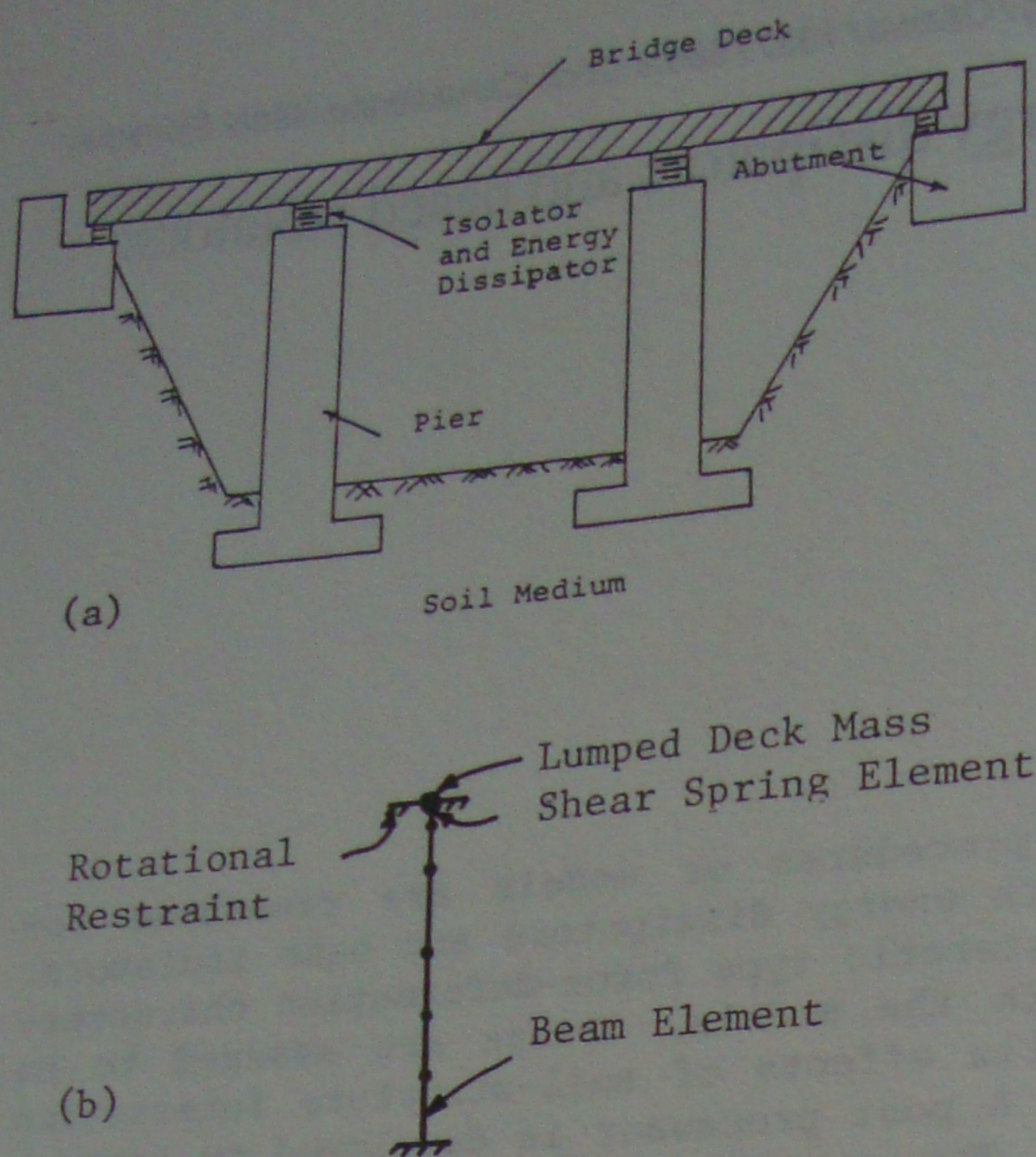


Fig. 1 (a) Bridge with energy dissipator and base isolator (b) finite element idealization (first model)

action phenomenon in base isolated bridges.

The interaction between a structure and surrounding soil can be viewed as a two-way phenomenon. First, the underlying soil medium affects the structural response, and second, the structure itself can modify the site response, in some cases. Most of the previous studies reported in the literature (see e.g. Valera et al., 1977; Tajirian et al., 1984; Ardhanareeswaran, 1984) appear to be concerned with the former aspect of soil-structure interaction only. In recent years, some studies pertaining to the latter aspect and applicable to nuclear power plant structures have been reported. No similar work is in evidence for bridges with energy dissipators and base isolators. Thus, the topic discussed in this paper involved a relatively new area of research significant to earthquake resistant design of bridges.

### 3 FE MODEL WITHOUT SOIL-STRUCTURE INTERACTION EFFECTS

In the present study, the finite element technique is used for evaluating earth-

quake response of bridges with energy dissipators and base isolators. Figure 1(a) shows a typical schematic diagram of a deck-girder bridge. The finite element idealization for the case without soil-structure interaction effects is depicted in Figure 1(b). Because the decks are relatively rigid in the longitudinal direction and the flexural modes of the deck do not contribute significantly to the total seismic response of such bridges, when subjected to horizontal support acceleration, they can be modeled effectively with an equivalent pier and a (contributory) deck mass at the top.

#### 3.1 Idealization of pier

The pier is discretized using conventional beam elements having two displacement and two rotational degrees-of-freedom (DOF) per element. The rotational DOFs are assigned zero mass and they are eliminated from the assembled system matrices, using a static condensation procedure, to avoid singularity problems which might otherwise arise. The pier is assumed to be linearly elastic during the loading process.

#### 3.2 Idealization of energy dissipator and base isolator

The energy dissipator and base isolator are modeled as a shear spring with two (lateral) displacement DOFs, one at each end. The force deflection characteristics of the base isolator in the lateral direction is assumed to be bilinear as shown in Figure 2. The following parameters are used to express its deformability characteristics:

$K_u$  = initial lateral stiffness

$K_d$  = post-elastic lateral stiffness

$Q_d$  = lateral characteristics strength

$U_{el}$  = maximum elastic displacement

The mass of the base isolator is assumed to be negligible (zero). In addition, the following assumptions are made:

1. The base isolator behaves identically in tension as in compression.
2. Loading and unloading stiffnesses

are equal in magnitude.

3. Elastic damping is expressed in terms of frequency computed based on  $K_u$ .

The stiffness and mass matrices of the base isolator are given in Figure 3. The stiffness coefficient  $K_b$  is assigned a value of  $K_u$  or  $K_d$  depending on whether the element  $u$  is undergoing loading or unloading and on the magnitude of lateral displacement. In the post-elastic range, damping is approximated by nonlinear hysteretic type constitutive relations (see Figure 2).

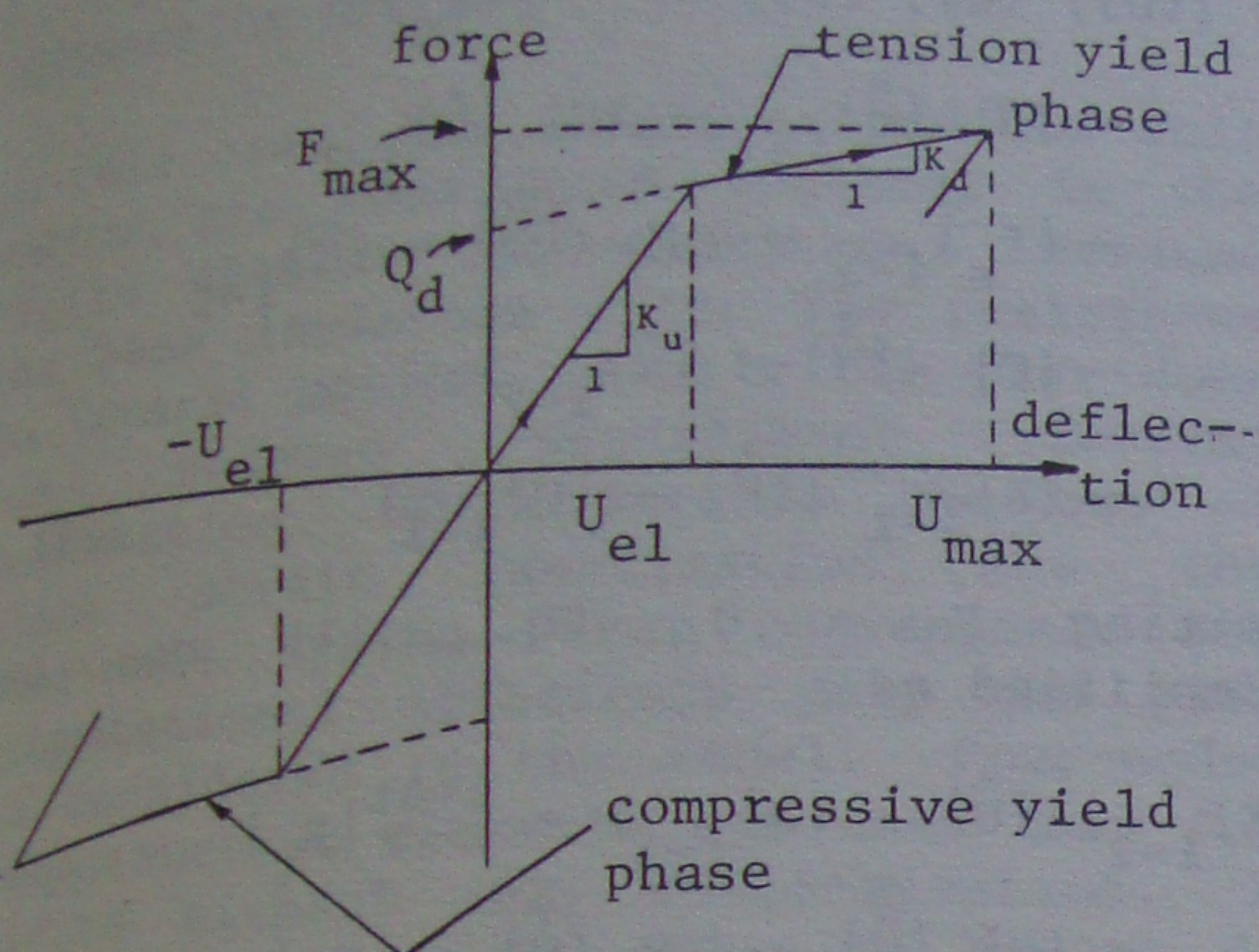
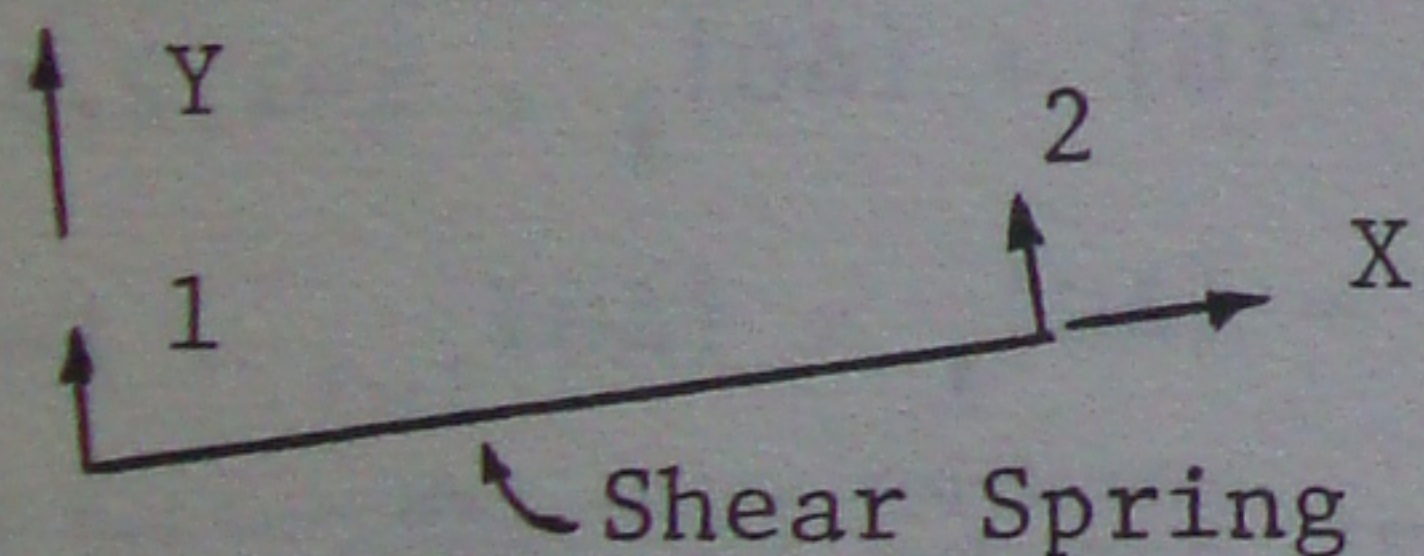


Fig. 2 Idealized force-deflection characteristics of base isolator-cum-energy dissipator



$$[K]_e = \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix} \quad [M]_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Fig. 3 Element matrices for base isolator (first model)

### 3.3 Equation of motion and solution procedure

In the simplified finite element procedure/model described above, non-linearity is caused by the energy dissipator and base isolator alone. The seismic excitation is applied in the

form of ground acceleration at the base of the pier. The equation of motion of the overall system is expressed in an incremental form as follows:

$$[M]^{t+\Delta t} \{\ddot{U}\} + [C]^{t+\Delta t} \{\dot{U}\} + {}^t[K] \{\Delta U\} = {}^{t+\Delta t}\{R_t\} - {}^t\{f_s\} \quad (1)$$

where

$[M]$  = Mass matrix

$[C]$  = Damping matrix

$[K]$  = Stiffness matrix

$\{U\}$  = Nodal displacement vector relative to support

$\Delta t$  = Time interval for integration

$$\{\Delta U\} = {}^{t+\Delta t}\{U\} - {}^t\{U\} \quad (2)$$

$${}^{t+\Delta t}\{R_t\} = -[M] \{X\} {}^{t+\Delta t} \ddot{U}_g \quad (3)$$

$\{f_s\}$  = Internal forces

$\{X\}$  = Pseudostatic displacement vector

$\ddot{U}_g$  = Ground acceleration

The superscripts on the left of the quantities denote the time and the over-dots represent derivative with respect to time. Pseudostatic displacement vector  $\{X\}$ , as described by Clough and Penzien (1975), defines the displacement along different degrees-of-freedom due to a unit support displacement.

The incremental equations of motion, Eq. (1), are solved using a step-by-step time integration procedure. The Newmark's constant average acceleration scheme (Bathe and Wilson, 1975) is used here for time integration.

### 3.4 Equilibrium iteration

The equilibrium equation used in Newmark's method (Bathe and Wilson, 1975) may not yield accurate results if there is a sudden change in stiffness/damping characteristics of the system, or if the force-displacement curve shows a high degree of nonlinearity (see Figure 2). In nonlinear dynamic analysis, it is important that the dynamic equilibrium is satisfied as accurately as possible at every time station. In this study a popular

iterative procedure, called Newton-Raphson scheme, is used for equilibrium iteration. A brief description of the iterative procedure adopted herein is given in this section.

Using the definition of incremental displacement vector  $\{\Delta U\}$ , Eq. (2), the global equations of dynamic equilibrium can be expressed as

$$[M] {}^{t+\Delta t}\ddot{\{U\}} + [C] {}^{t+\Delta t}\dot{\{U\}} = {}^{t+\Delta t}\{R_t\} - {}^{t+\Delta t}\{f_s\} \quad (4)$$

The nonlinear function  $F$ , whose zeros are sought in this case can be derived from Eq. (4) by transposing the terms as

$$F[\{\hat{U}\}, \{\hat{\dot{U}}\}, \{\hat{\ddot{U}}\}] = {}^{t+\Delta t}\{R_t\} - [M] {}^{t+\Delta t}\{\hat{\ddot{U}}\} - [C] {}^{t+\Delta t}\{\hat{\dot{U}}\} - {}^{t+\Delta t}\{f_s\} \quad (5)$$

where  $\hat{U}$ ,  $\hat{\dot{U}}$  and  $\hat{\ddot{U}}$  are dummy arrangements of  $F$  and they correspond to system displacement, velocity and acceleration response, respectively, at time  $(t+\Delta t)$ . The objective is to solve the set of equations

$$F[\{\hat{U}\}, \{\hat{\dot{U}}\}, \{\hat{\ddot{U}}\}] = 0 \quad (6)$$

The general approach in the Newton-Raphson scheme is to estimate an initial solution and then to improve upon the initial estimate iteratively.

Let  $\{U\}_{i-1}$ ,  $\{\dot{U}\}_{i-1}$ ,  $\{\ddot{U}\}_{i-1}$  be the response value at time  $t+\Delta t$  at the end of  $(i-1)$ th iteration. (In the discussion that follows, all the response values correspond to time  $(t+\Delta t)$ , unless otherwise stated.) Expanding  $F[\{\hat{U}\}, \{\hat{\dot{U}}\}, \{\hat{\ddot{U}}\}]$  by using Taylor series in the neighborhood of  $\{U\}_{i-1}$ ,  $\{\dot{U}\}_{i-1}$  and  $\{\ddot{U}\}_{i-1}$ , and neglecting the higher order terms we can obtain

$$F[\{\hat{U}\}, \{\hat{\dot{U}}\}, \{\hat{\ddot{U}}\}] = {}^{t+\Delta t}\{R_t\} - [M] \{\ddot{U}\}_{i-1} - [C] \{\dot{U}\}_{i-1} - \{f_s\}_{i-1} + [D(F, \hat{U})] (\{\hat{U}\} - \{U\}_{i-1}) + [D(F, \hat{\dot{U}})] (\{\hat{\dot{U}}\} - \{\dot{U}\}_{i-1}) + [D(F, \hat{\ddot{U}})] (\{\hat{\ddot{U}}\} - \{\ddot{U}\}_{i-1}) \quad (7)$$

where  $D[f, x]$  = partial derivative of  $f$  with respect to  $x$ , and  $\{f_s\}_{i-1}$  = internal force corresponding to  $\{U\}_{i-1}$ . The derivatives in Eq. (7) are taken at  $\{U\}_{i-1}$ ,  $\{\dot{U}\}_{i-1}$ , and  $\{\ddot{U}\}_{i-1}$ . It should

be noted that

$$[D(F, \hat{U})] = -[M], \quad (8)$$

$$[D(F, \hat{\dot{U}})] = -[C], \quad \text{and} \quad (9)$$

$$[D(F, \hat{\ddot{U}})] = [K]_{i-1} \quad (10)$$

Equations (8) through (10) assume that the external load  $\{R_t\}$  is not affected by the response of the system. Substituting for the derivatives in Eq. (7) from Eqs. (8), (9), and (10), the following expression can be obtained.

$$F[\{\hat{U}\}, \{\hat{\dot{U}}\}, \{\hat{\ddot{U}}\}] = {}^{t+\Delta t}\{R_t\} - [M] \{\ddot{U}\}_{i-1} - [C] \{\dot{U}\}_{i-1} - \{f_s\}_{i-1} - [M] (\{\hat{\ddot{U}}\} - \{\ddot{U}\}_{i-1}) - [C] (\{\hat{\dot{U}}\} - \{\dot{U}\}_{i-1}) - [K]_{i-1} (\{\hat{U}\} - \{U\}_{i-1}) \quad (11)$$

setting  $F = 0$ , Eq. (11) can be simplified as

$$[K]_{i-1} (\{\hat{U}\} - \{U\}_{i-1}) = {}^{t+\Delta t}\{R_t\} - [M] \{\hat{\ddot{U}}\} - [C] \{\hat{\dot{U}}\} - \{f_s\}_{i-1} \quad (12)$$

$\{\hat{U}\}$ ,  $\{\hat{\dot{U}}\}$ ,  $\{\hat{\ddot{U}}\}$  form the new improved solution set at the end of  $i$ th iteration. In other words,  $\{U\}_i = \{\hat{U}\}$ ,  $\{\dot{U}\}_i = \{\hat{\dot{U}}\}$ , and  $\{\ddot{U}\}_i = \{\hat{\ddot{U}}\}$ . Defining  $\{U\}_i$  and  $\{\Delta^2 U\}_i$  as

$$\{U\}_i = {}^t\{U\} + \{\Delta U\}_i, \quad i=1,2,3,\dots \quad (13)$$

$$\{\Delta^2 U\}_i = \{\Delta U\}_i - \{\Delta U\}_{i-1}, \quad i=1,2,3,\dots \quad (14)$$

Eq. (12) can be rewritten as

$$[K]_{i-1} \{\Delta^2 U\}_i = {}^{t+\Delta t}\{R_t\} - [M] \{\ddot{U}\}_i - [C] \{\dot{U}\}_i - \{f_s\}_{i-1} \quad (15)$$

Since  $\{\dot{U}\}_i$  and  $\{\ddot{U}\}_i$  are not known a priori, Eq. (15) cannot be solved in this form. This problem is overcome by expressing  $\{\dot{U}\}_i$  and  $\{\ddot{U}\}_i$  in terms of  ${}^t\{U\}$ ,  $\{\Delta U\}_{i-1}$  and  $\{\Delta^2 U\}_i$ . This would give a system of equations in which  $\{\Delta^2 U\}_i$  is the only unknown. Knowing  $\{\Delta^2 U\}_i$ , other system response  $\{\Delta U\}_i$ ,  $\{U\}_i$ ,  $\{\dot{U}\}_i$ ,  $\{\ddot{U}\}_i$  can be obtained easily (see Eqs. (13) and (14)). Further details of the iterative procedure are given by Ardhanareeswaran (1984).

### 3.5 Local extrema

In the dynamic problem under consideration, it is necessary to identify the peak displacement at which the system (i.e., base isolator) starts unloading from tension to compression or vice versa. In the present study the peak displacement is assumed to occur when the velocity response of the base isolator equals zero. Details could not be given here because of space limitation.

### 4 FE MODEL WITH SOIL-STRUCTURE INTERACTION EFFECTS

In this case, the structure and the soil medium are considered together in the finite element discretization. Because the base isolators offer low resistance to lateral motion, usually the pier does not undergo significant lateral deformation. This permits the use of plane strain idealization for the pier-soil system. For convenience and computational efficiency only one pier is considered in the model. Four-noded isoparametric elements are used here for finite element (FE) discretization. A typical FE mesh is shown in Figure 4. Both the pier and the soil medium are considered to be linearly elastic and a constant Rayleigh damping matrix (Seed and Hwang, 1975) is used to estimate the damping characteristics. Damping ratios assumed in soil and structure are, respectively, 10% and 5% of critical damping.

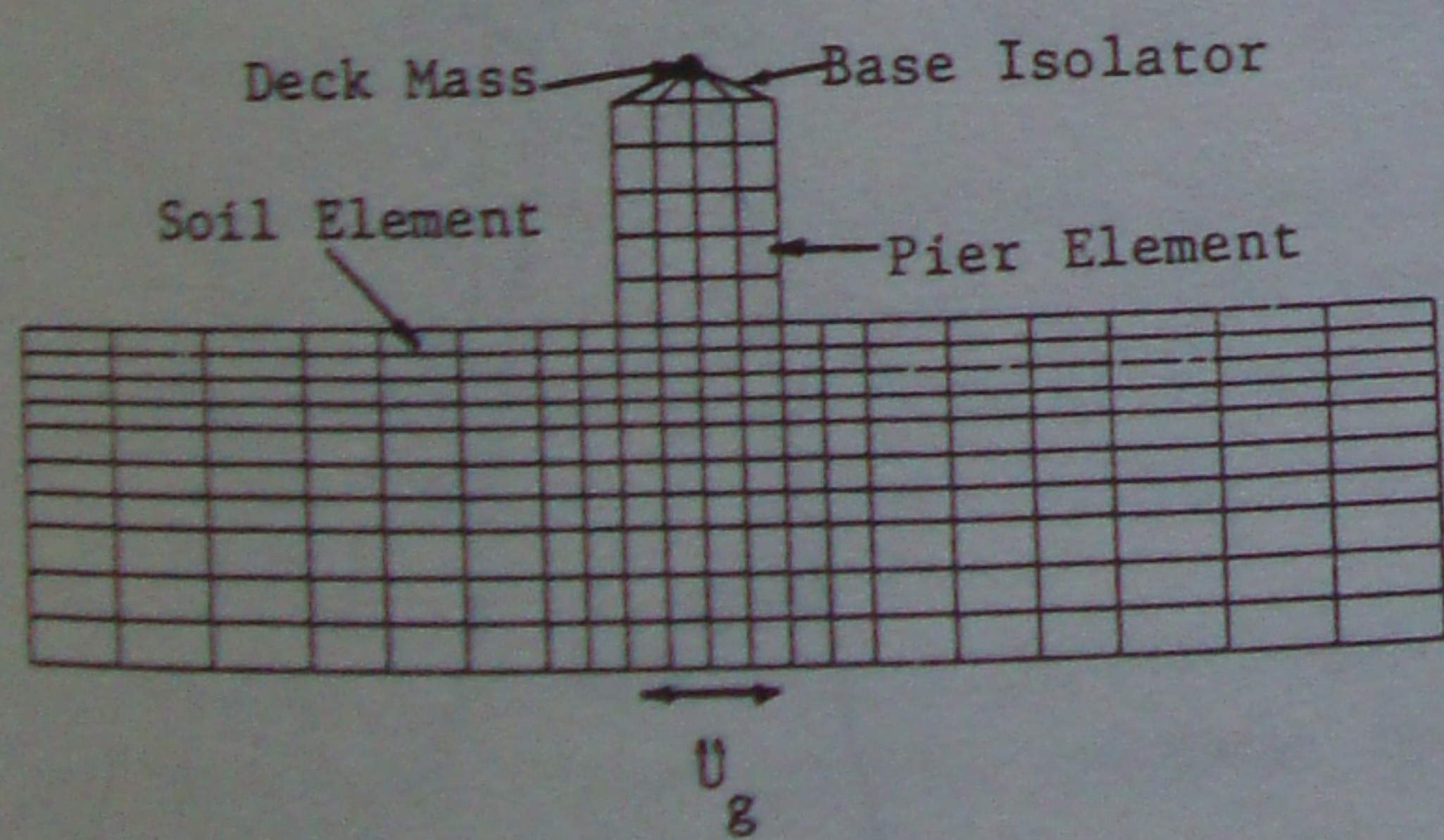


Fig. 4 Finite element mesh (second model)

The deck mass is assumed to act as a rigid body and hence modeled as a lumped mass (see Figure 4). The deformability characteristics of the base isolator are modeled in a manner analogous to that described in Section 3 except that two additional (vertical) degrees-of-freedom are considered for this case. The base

isolator elements are considered to be geometrically dimensionless, (i.e., zero mass). The element stiffness matrix for a typical element is given in Figure 5. A procedure similar to that discussed in Section 3 is used for solving the incremental nonlinear equations of motion.

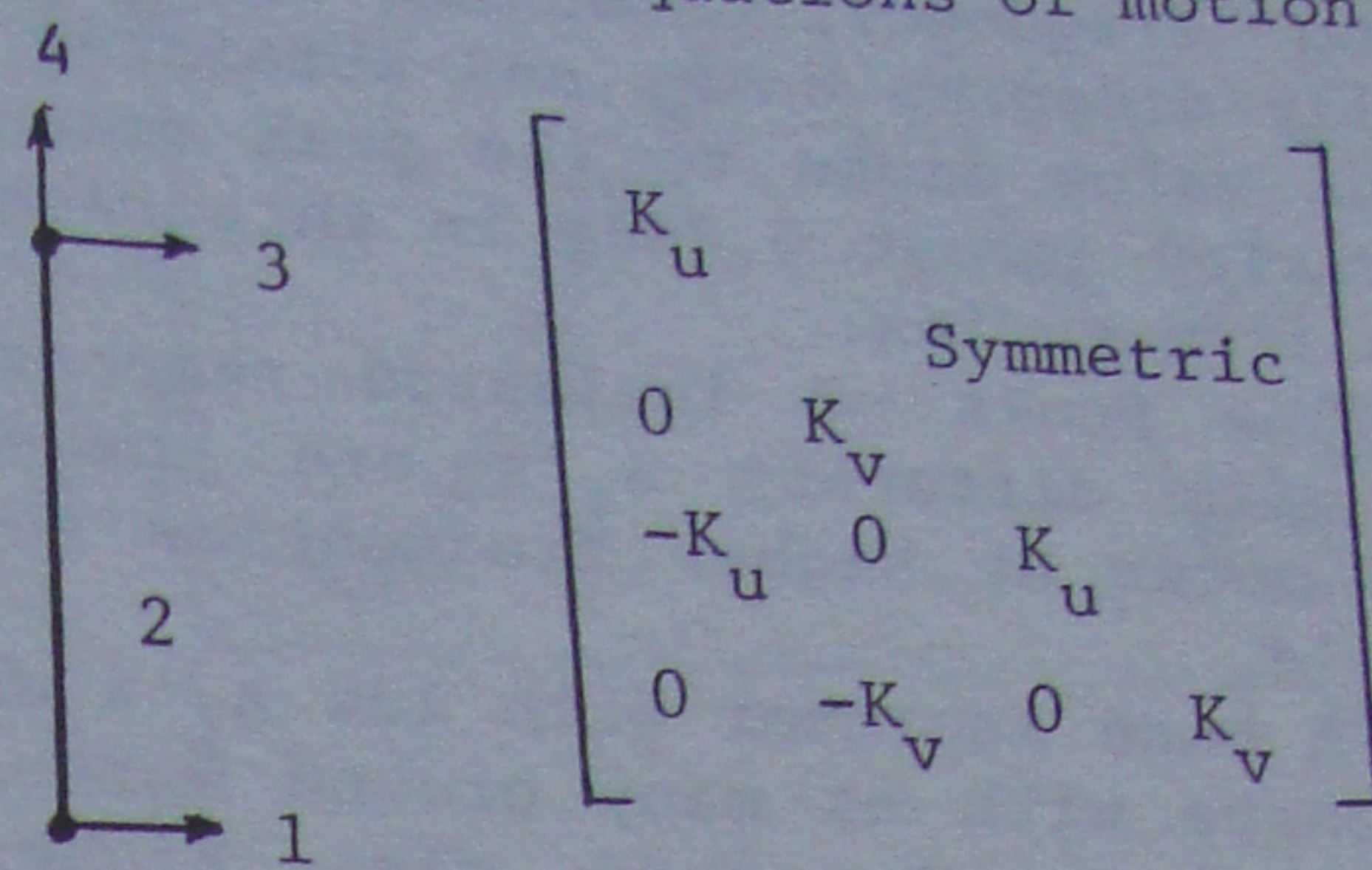


Fig. 5 Stiffness matrix of base isolator element (second model)

### 5 NUMERICAL RESULTS

#### 5.1 Model without soil-structure interaction effects

For convenience of presentation of numerical results two symbols are introduced herein: (i) characteristic strength ratio ( $C_s$ ), and (ii) characteristic stiffness ratio ( $A_1$ ).  $C_s$  is defined as the ratio of characteristic strength of the base isolator to weight of the deck, where  $A_1$  is defined as the ratio of the post elastic stiffness of the base isolator to its initial stiffness (i.e.,  $A_1 = K_d/K_u$ ). Parametric studies were conducted to investigate the influence of  $C_s$  and  $A_1$  on the following system responses:

1. Peak total acceleration, expressed in terms of acceleration due to gravity, and

2. Peak displacement ratio, expressed as a ratio of peak relative displacement to maximum elastic displacement.

The material and geometric properties used in the parametric study are given by Ardhanareeswaran (1984). Typical results in the form of spectral response (acceleration), pertaining to parameter  $A_1$  are presented below.

In this parametric study, values of  $C_s$  and the damping ratio ( $D$ ) were kept constant at 0.05. A frequency range of 0.2-20 Hz was chosen. The first four seconds of the ground acceleration recorded at El Centro (May 1940 N-S component) was used as ground accelera-

tion input. The results are shown in Figures 6 and 7. The following features are worth noting:

(i) All inelastic response curves show the same general pattern. The peak total acceleration increases with increasing frequency, reaching a peak, gradually dropping down, and attaining a constant value close to the peak ground acceleration (about 0.348g in this case) at high frequencies.

(ii) The peaks of inelastic response curves with different  $A_i$ 's are phased out and occur at higher frequencies for lower  $A_i$ 's.

(iii) A base isolator with low  $A_i$ 's do not resonate with as many ground motion frequencies as the ones with higher  $A_i$ 's. The spectral response curves with low  $A_i$ 's are relatively smooth compared to those with high  $A_i$ 's. Response curves for high  $A_i$ 's show multiple peaks and appear closer to elastic spectrum. Physically, this means that base isolator with lower post elastic stiffness isolate the deck better from irregular ground motions. In other words, to achieve better isolation characteristics, lower values of  $A_i$  should be recommended. This is, however, at the cost of increased peak displacement ratio.

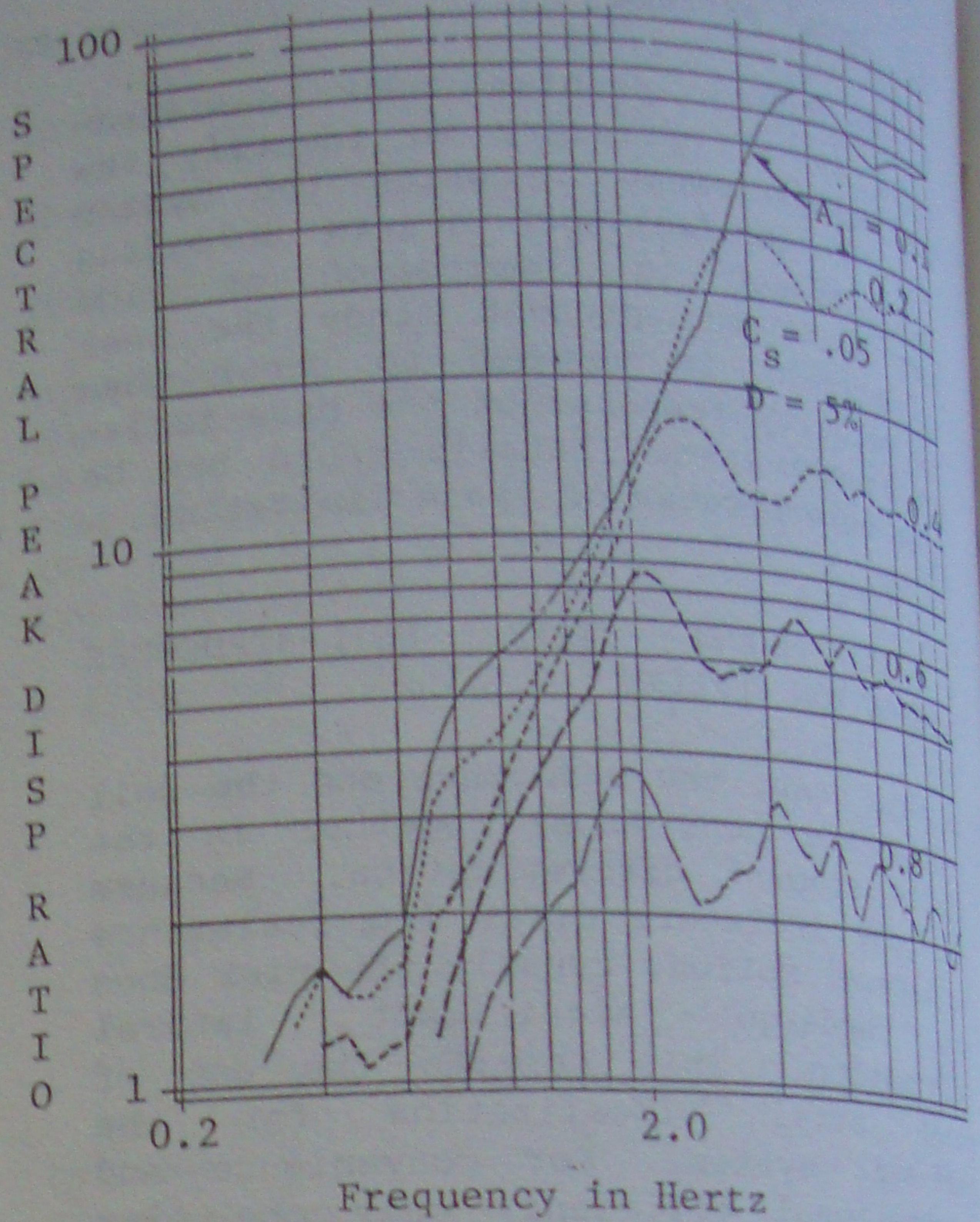


Fig. 7 Inelastic displacement spectra for different  $A_i$  (first model)

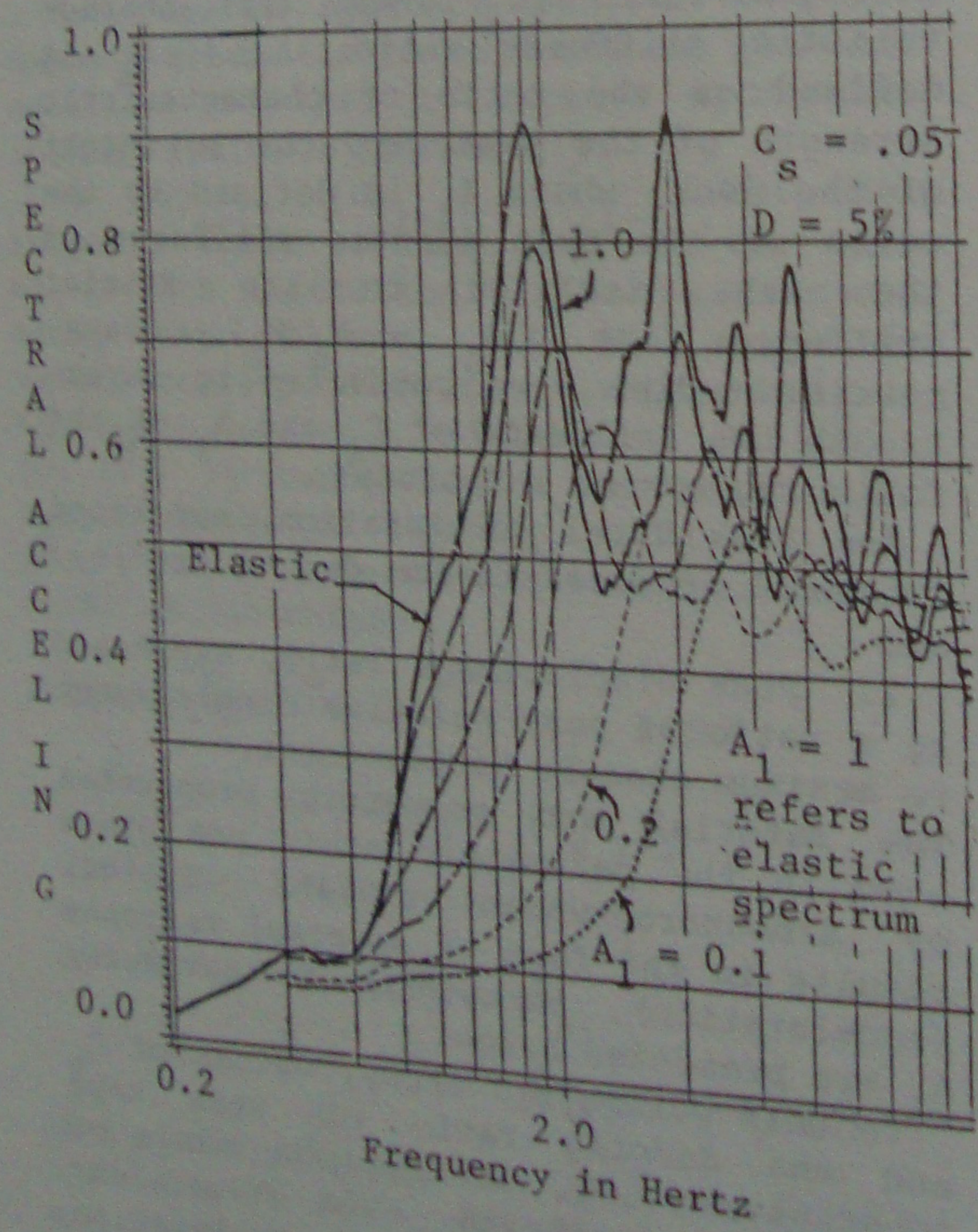


Fig. 6 Inelastic acceleration spectra for different  $A_i$  (first model)

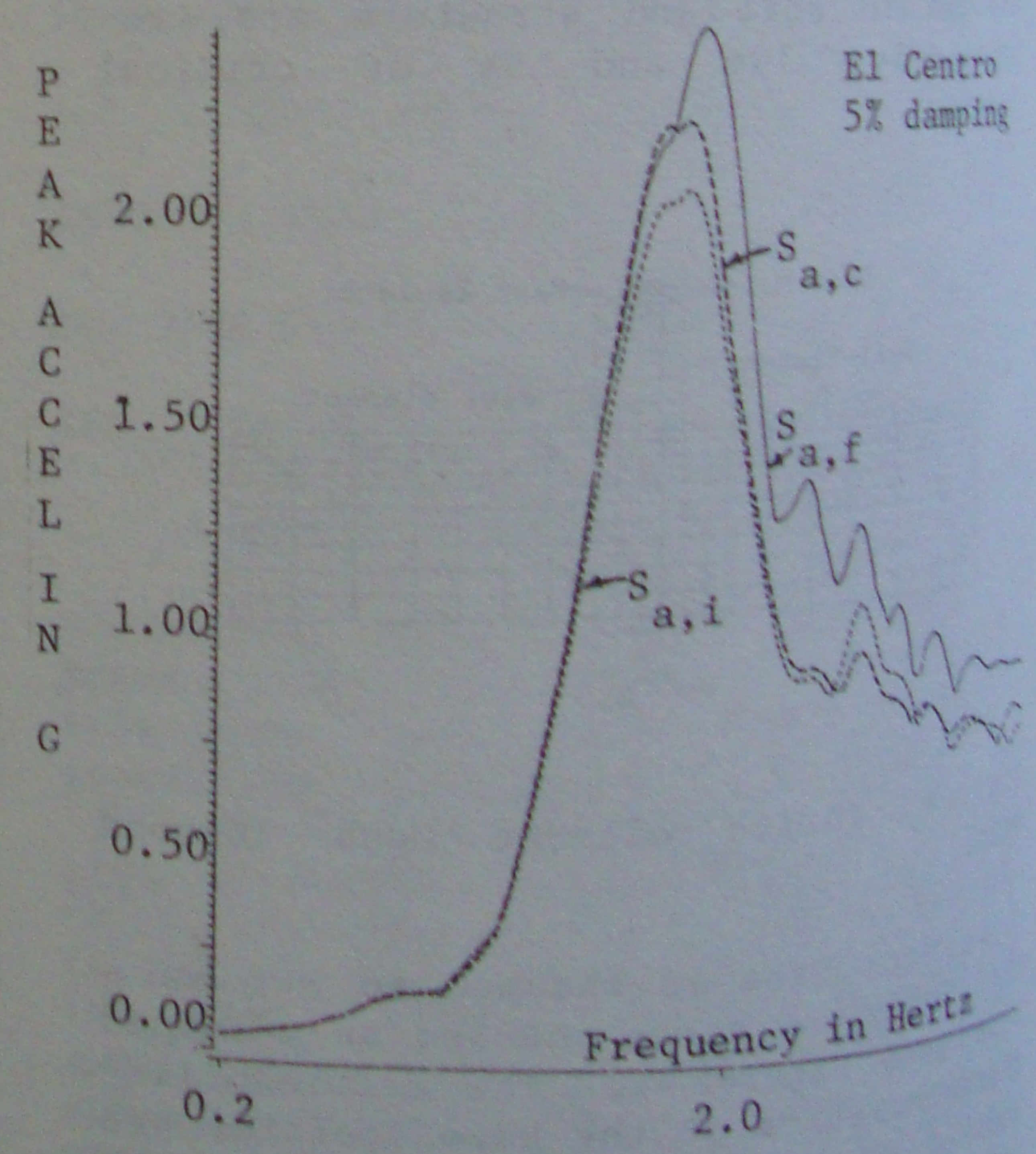


Fig. 8 Spectral acceleration response (second model)

## 5.2 Model with soil-structure interaction effects

A medium site with a shear wave velocity of 548.6 m/s was selected for investigating the earthquake response of a base isolated pier including soil-structure interaction effects. The pier considered herein is a massive, wall type pier geometrically similar to that used by Toki et al. (1981) in their analysis. The deck mass is isolated from the pier-top by means of a commonly used base-isolator-cum-energy dissipator. Pertinent material properties are given in Table 1 (Zaslavsky and Wight 1975, 1983). Ground acceleration input is identical to that used in the previous case (see Section 5.1)

Table 1. Pertinent material properties used.

Source	$V_s^*$	$\nu$	$\gamma$	D
Structure	1600	0.167	2.4	5%
Soil	548.6	0.4	1.8	10%

Deck weight = 1,920 t;  $K_u = 7,728$  t/m,  
 $A_1 = 0.16$ ;  $C_s = 0.05$

\*  $V_s$  = shear wave velocity (m/s);  $\nu_3$  = Poisson's ratio;  $\gamma$  = unit weight (t/m<sup>3</sup>); and D = damping ratio

The following symbols relate to the spectral acceleration curves generated in this investigation.

$S_{a,f}$  = spectral acceleration based on free-field motions on surface

$S_{a,c}$  = spectral acceleration based on motions at a point close to structure, and

$S_{a,i}$  = spectral acceleration based on motions at a point common to soil and structure.

Figure 8 shows the spectral acceleration curves for far and near fields (i.e.,  $S_{a,f}$ ,  $S_{a,c}$ ,  $S_{a,i}$ ). It is observed that, in general, for a given frequency,  $S_{a,c}$  and  $S_{a,i}$  are less than  $S_{a,f}$ . Although  $S_{a,c}$  and  $S_{a,i}$  both pertain to two close by points in the system, they depict somewhat different values. This may be caused by the presence of interfaces between soil and pier having dissimilar properties.

The significance of Figure 8 can be

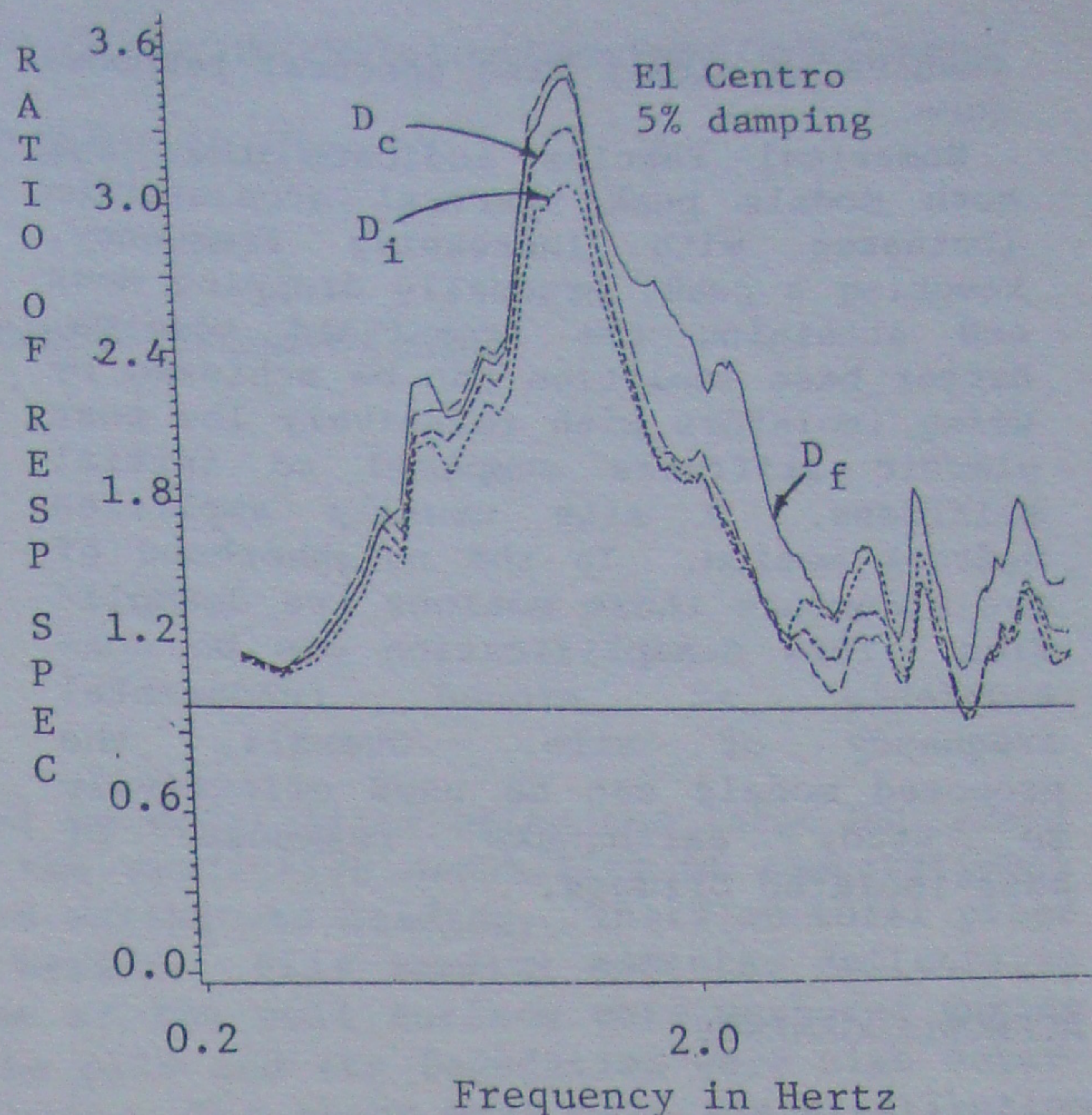


Fig. 9 Ratio of spectral response curves (second model)

better understood if the results are presented (see Figure 9) in the form of ratio of spectral response (RSR) (i.e.,  $D_x = S_{a,x}/S_{a,r}$ , where  $x = c, f, i$ ,  $S_{a,r}$  being the spectral acceleration based on bed-rock acceleration). Note that the peak values of  $D_f$ ,  $D_c$  and  $D_i$  are approximately 3.6, 3.3<sup>f</sup> and 3.1, respectively. The presence of structure is seen to dampen the response by as much as 16%. Also, in general,  $D_c < D_f$  over the frequency range considered. Similar results for a soft site are given by Ardhanareeswaran et al. (1985).

## 6 SUMMARY AND CONCLUDING REMARKS

Two finite element based models have been presented in this paper for evaluating earthquake response of deck-girder bridges with base isolators-cum-energy dissipators. The second model accounts for the soil-structure interaction effects while the first model does not. Emphasis is given to realistically modeling the nonlinear and hysteretic type force-deformation characteristics of the base isolators. The resulting incremental equations of motions are solved in the time domain using an implicit time integration scheme. A Newton-Raphson type iterative procedure is used for equilibrium iterations. A post processor is developed for presenting

results in widely used spectral response form.

Numerical results indicate that for both models peak spectral acceleration increases with increasing frequency, reaching a peak, gradually dropping down and attaining the free-field spectra. Better base isolation can be achieved by using isolators with relatively low post elastic stiffness compared to initial stiffness. A site usually amplifies bedrock motion. In the neighborhood of the structure these motions are deamplified. This deamplification can be considerable at around fundamental frequency of site. Overall, the proposed models can be used effectively to study earthquake response of base-isolated bridges.

#### ACKNOWLEDGEMENTS

Professor L.R.L. Wang of Old Dominion University, Norfolk, Virginia, supervised part of the work reported in this paper. His assistance is gratefully acknowledged. Computing facilities of the University of Oklahoma, Norman were used for all computations. Ms. Betty Craig, a staff of Civil Engineering and Environmental Science, typed the manuscript.

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